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Emergence of product differentiation from consumer heterogeneity and asymmetric information

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Abstract. We introduce a fully probabilistic framework of consumer product choice based on quality assessment. It allows us to capture many aspects of marketing such as partial information asymmetry, quality differentiation, and product placement in a supermarket.

PACS. 89.65.Gh Economics; econophysics, financial markets, business and management – 89.75.-k Complex systems

1 Introduction

The interests of vendors and customers seem antagonistic a priori, the former aiming at decreasing quality and increasing price, whereas the latter wishing exactly the opposite. The situation is fortunately more complex, the interests of both sides being sometimes compatible. Intuitively, a vendor may sell more items by increasing their perceivable quality, making everybody happier. But the situation is more subtle because of asymmetric information: the vendor knows much better than his prospective customers the real quality of his products. In Akerlof's famous Lemon Problem, the customers have no means to assertain the quality of products, which leads to a no-trade paradox [1]. When the customers are better equipped, optimal quality emerges [2–4]. One of the main issues is to understand under which conditions a manufacturer should diversify his production. Economics literature has approached this problem mainly with the help of utility functions. Several aspects have been studied, among them optimal quality-based product differentiation [5], firm competition by quality [6] and by price [7], the relation between product quality and market size [8], etc. (see [9,10] for a review).

We assume that customers' decisions, while influenced by perceived properties of the products, are probabilistic in nature. Using a probabilistic consumer choice framework makes it possible to avoid utility functions and hence our model can be understood as an alternative to the usual utility-function approach. For other alternatives, known as models of discrete or probabilistic choice, which still use utility theory and yet they are probabilistic see [11–13]. In our work we take the point of view of a monopolistic vendor faced to consumers deciding to buy one of his products according to their perception of its quality. The resulting complex complex system with one vendor, several product variants, and many heterogeneous buyers, is investigated by numerical techniques.

The paper is organized as follows. In Section 2 we introduce our framework and determine the optimal quality of a single product proposed to homogeneous or heterogeneous customers. In Section 3 we examine the conditions under which a vendor should segment the market by manufacturing several products of different quality. In Section 4 we allow the vendor to optimize the price as well. We leave to the appendices a deeper discussion of our assumptions and more technical results on the economics of spamming.

2 Single product

We assume that the only difference between products lies in their quality $Q \geq 0$ which is therefore the main quantity of interest here.[1] With a suitable choice of units, one can write the profit of a vendor per item sold as $1-F(Q)$ where F is an increasing function and $F(1) = 1$. For the sake of

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[1] One can also build a model starting from the tastes of the buyers as in [14].

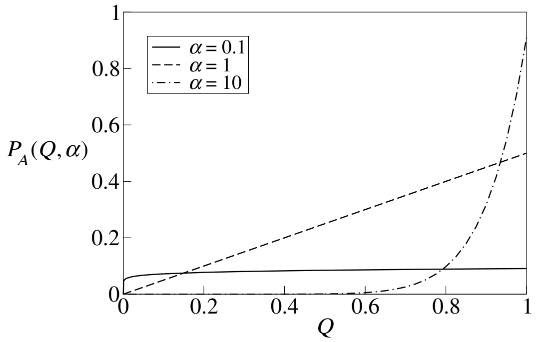

Fig. 1. The acceptance probability $P_A(Q, \alpha)$ for various values of the acceptance parameter α.

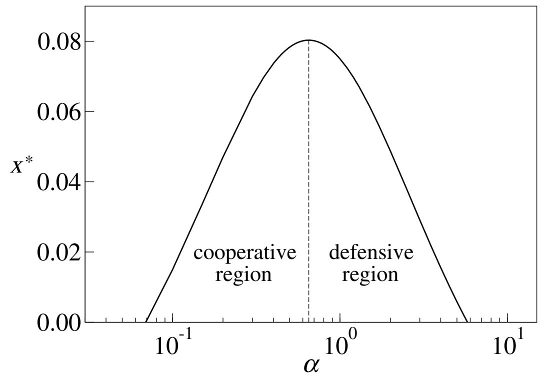

Fig. 2. Optimal vendor's profit per buyer x^* as a function of α for $z = 0.05$.

simplicity, we take $F(Q) = Q$; $F(Q) \propto Q^2$ is also found in the literature but does not alter qualitatively our results. While Q could in principle be greater than one, a vendor would never choose it, hence our analysis is restricted to $Q \in [0, 1]$.

We assume that a customer buys a product of quality Q with acceptance probability $P_A(Q)$. While there are many possible choices, e.g. those of references [9,15] or piece-wise linear functions as in reference [4], we shall mainly use

$$P_A(Q, \alpha) = \left(1 - \tfrac{1}{\alpha+1}\right) Q^\alpha \qquad (\alpha \geq 0) \tag{1}$$

where $\alpha > 0$ is the acceptance parameter: for small α, P_A is mostly flat, resulting in a lack of quality discrimination; as α grows, the core of P_A shifts towards higher quality, which reflects enhanced discrimination abilities (see Fig. 1). We will use the shorthands "ignorant" for buyers with a small α and "informed" for those with a large α; an ignorant buyer is quite likely to reject even a perfect product. Since for $\alpha > 0$ and $Q \in [0; 1]$ is $P_A(Q, \alpha) < 1$, by equation (1) we implicitly assume that for the considered product there are substitutes which can satisfy needs of consumers.

If there are N buyers with acceptance parameters α_i ($i = 1, \ldots, N$), faced with a single product of quality Q, the vendor's expected profit X is

$$X(Q) = (1 - Q) \sum_{i=1}^{N} P_A(Q, \alpha_i) - Z \tag{2}$$

where Z represents the fixed part of production costs due, for instance, to the initial investment needed to setup the manufacturing plant. Assuming that N is large, the fluctuations of X can be neglected. The structure of this expression is similar to the profit function introduced in [9]. Since $X'(0) > 0$, $X'(1) < 0$, and $X(Q)$ is a continuous function, there is at least one Q maximizing X in $(0, 1)$.

In the following we take the point of view of the vendor and hence optimize his expected profit X.

2.1 Homogeneous population

When there is only one type of buyers, the expected profit simplifies to $X(Q) = N(1-Q)P_A(Q, \alpha) - Z$ which reaches its maximum at

$$Q^*(\alpha) = \frac{\alpha}{\alpha + 1}\cdot \tag{3}$$

Expectedly, $Q^*(\alpha)$ increases when the buyers have a sharper eye. The total optimal profit reads

$$X^*(\alpha) = N \frac{\alpha^{\alpha+1}}{(\alpha + 1)^{\alpha+2}} - Z. \tag{4}$$

In Figure 2 we report the expected optimal profit per customer $x^*(\alpha)\colon = X^*(\alpha)/N$ as a function of α for $z\colon = Z/N = 0.05$. When $z > 0$, a vendor only makes a profit when the quality is not too high or too low. Accordingly $x^*(\alpha)$ has a maximum at $\alpha_0 \approx 0.65$. Therefore, if the vendor cannot easily change Q, he should target a population with Q^*, or strive to modify the abilities of his prospective customers to detect quality, thereby increasing his profit. When $0 \leq \alpha < \alpha_0$), both the consumers and the vendor benefit from an increase in Q; we shall call it the cooperative region. Reversely, when $\alpha > \alpha_0$ the vendor suffers from excessive quality detection abilities of his customers; he could try a confusing marketing campain or rebranding so as to lower their abilities – this is the defensive region. A similar behaviour has been observed in [16]. In our case, the fact that the cooperative region is much smaller than the defensive region is a consequence of the shape of P_A. For instance, when the prefactor in $P_A(Q)$ changes from $1-(\alpha+1)^{-1}$ to $1-(\alpha+1)^{-1/3}$, the size of the cooperative region increases significantly.

2.2 Heterogeneous buyers

Heterogeneity brings in more surprises. Let us split the population into two groups, group $i = 1, 2$ consisting of N_i buyers with acceptance parameter α_i; the proportion of group i is denoted by $c_i\colon = N_i/N$. The vendor's expected profit reads

$$X(Q) = N(1 - Q)\big[c_1 P_A(Q, \alpha_1) + c_2 P_A(Q, \alpha_2)\big] - Z. \tag{5}$$

It is not possible to maximize X analytically. The result of numerical investigations is shown in Figure 3 as a function of c_2 for $\alpha_1 = 0.1$ (ignorant buyers) and various choices of

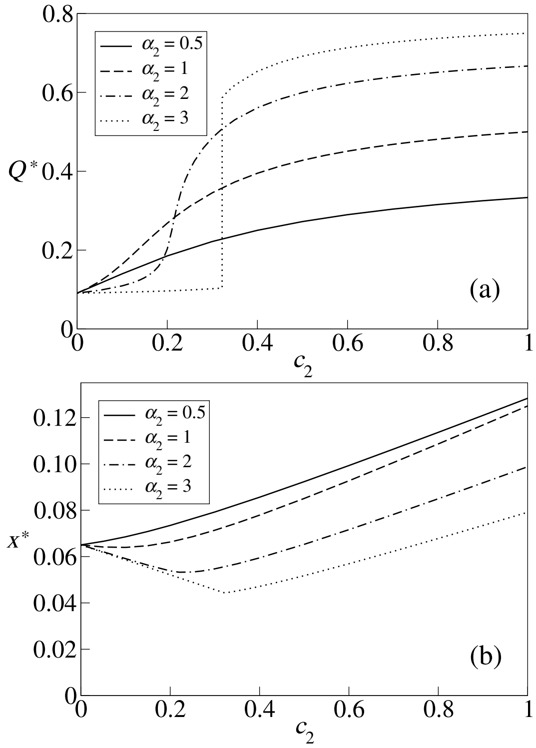

Fig. 3. Optimal product quality Q^* (left) and vendor's optimal profit x^* (right) versus proportion c_2 for various values of α_2; $\alpha_1 = 0.1$, $z = 0.01$.

α_2. As expected, as the proportion of informed buyers increases, Q^* grows. But a surprising behaviour is found for instance when $\alpha_2 = 3$: at $c_2 \approx 0.32$ the optimal quality changes discontinuously. This is because $X(Q)$ has two local maxima. While for small c_2 the small-Q peak yields the largest profit, its relative height decreases as c_2 increases; accordingly the discontinuous transition occurs when the heights of the two maxima are equal. In Figure 3 we also show the dependence of the optimal profit per buyer x^* on c_2. When group 2 has $\alpha_2 < \alpha_0$ (e.g., $\alpha_2 = 0.5$), adding people with more demands regarding quality is beneficial to the vendor (Eq. (4)) and X is an increasing function of c_2. By contrast, when $\alpha_2 > \alpha_0$ the optimal profit first decreases as almost nobody of group 2 will buy anything and does so as long as group 2 has less influence on Q^* than group 1. Then group 2 supercedes group 1 and imposes its quality demands; the discussion generalises to an arbitrary number of groups. In other words, when society is too heterogeneous, it is impossible to satisfy all buyer groups with one product.

3 Multiple products

Now we assume that the vendor displays M product variants of different quality, at equal prices for the sake of simplicity, and that each buyer buys at most one item. A purchase is a two-step process, as a shopper has also to decide on a variant. The choice is also assumed to be probabilistic: variant $m = 1, \ldots, M$ is chosen according to

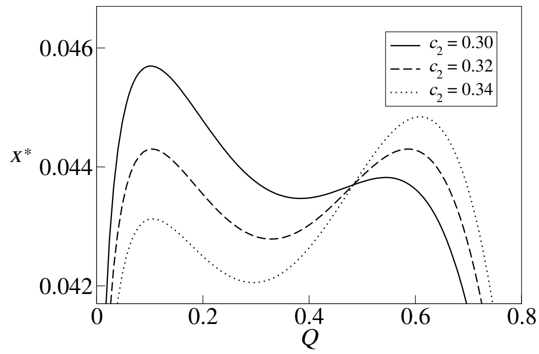

Fig. 4. Optimal profit per buyer x^* as a function of quality Q: one product, two groups of buyers ($\alpha_1 = 0.1$, $\alpha_2 = 3.0$). As c_2 increases, at $c_2 \approx 0.32$ the heights of the maxima are equal and a discontinuous change of the optimal quality occurs.

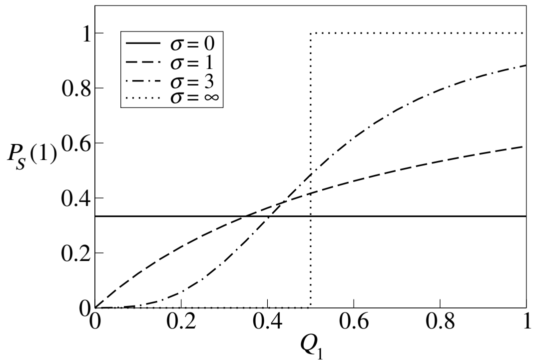

Fig. 5. The probability to select variant 1 as a function of its quality Q_1 for various values of the selection parameter σ. In total three variants are displayed, the qualities $Q_2 = 0.5$ and $Q_3 = 0.2$ are fixed.

$$P_S(m|\boldsymbol{Q}, \sigma) = \frac{Q_m^\sigma}{\sum_{m'=1}^{M} Q_{m'}^\sigma}. \tag{6}$$

Here $\sigma \in [0, \infty)$ quantifies the selection ability of a given buyer. When σ is large, the buyer almost surely selects the best variant; on the contrary when $\sigma = 0$, $P_S(m) = 1/M$ for all m, i.e., the buyer has no discerning power. Since P_S is normalized, each buyer purchases at most one item. Similar expressions appear in works on the influence of advertisement [15] and non-price competition [9], but other choices of functions would also be reasonable, such as exponentials as in the Logit model [13,17]. All $Q_{m'}^\sigma$ have equal weight in equation (6); Section 3.3 generalizes this expression in order to take into account the proportions of displayed items. Finally, a more complete discussion on the plausibility of P_S is given in Appendix A.

To summarize, the variant m with the quality Q_m is bought by buyer i with probability $P_S(m|\sigma_i, \boldsymbol{Q}) P_A(Q_m, \alpha_i)$. As a consequence, if the vendor displays M variants to N buyers, his expected profit is

$$X(\boldsymbol{Q}) = \sum_{m=1}^{M} (1 - Q_m) \left(\sum_{i=1}^{N} P_S(m|\boldsymbol{Q}, \sigma_i) P_A(Q_m, \alpha_i) \right) - MZ. \tag{7}$$

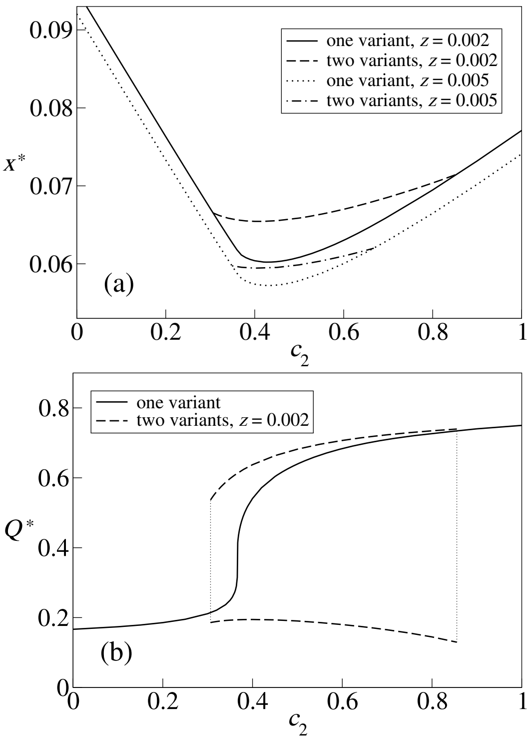

Fig. 6. (a) Optimal profits per buyer as a function of proportion c_2; two-variant profits x_2^* (broken and dashdot lines) are shown only when quality differentiation occurs. (b) Optimal quality as a function of proportion c_2, curves for the differentiated qualities Q_1^* and Q_2^* are only shown when $x_2^* > x_1^*$. Values of parameters are $\alpha_1 = 0.2$, $\alpha_2 = 3.0$, $\sigma_1 = 0.5$, and $\sigma_2 = 3.0$.

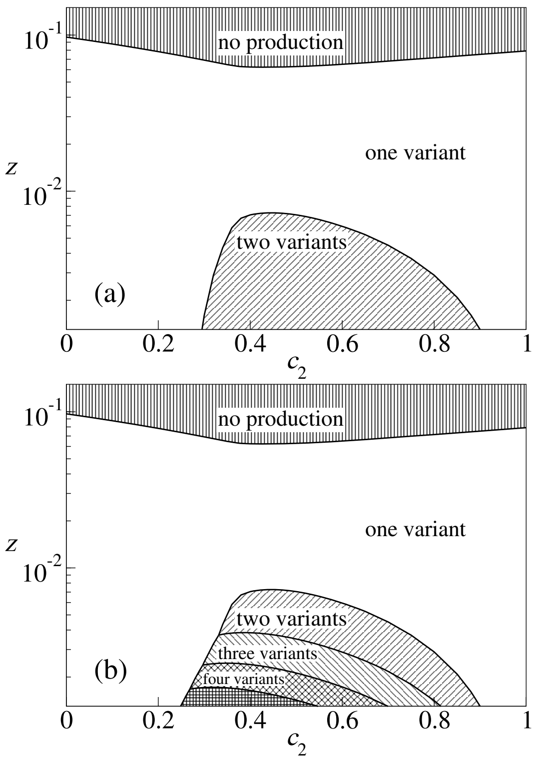

Fig. 7. Phase space (c_2, z) of optimal production: only $0, 1, 2$ variants are allowed (a) and without constraints on the maximal number of variants (b). Same parameters as in Figure 6.

This equation can be easily extended to account for special circumstances. For example, when Z is large, it may be profitable to produce one variant and achieve quality differentiation by artificially damaging a fraction of the production, e.g. by disabling some features [18]. In this case two variants with qualities $Q_1 > Q_2$ are displayed but the profit per item sold is only $1 - Q_1$ for both of them and the initial cost is reduced from $2Z$ to Z.

3.1 Quality differentiation

For the sake of simplicity, we focus on two groups of customers consisting of N_i members with acceptance parameter α_i and selection power σ_i $(i = 1, 2)$. The question is whether the vendor should display one or two products. In our framework, the answer is entirely determined by the respective optimal profit of each possibility, denoted by $X_1^*(Q)$ and $X_2^*(Q_1, Q_2)$.

Since manufacturing two products requires twice as much initial investment (by hypothesis), the region in which $X_2^* > X_1^*$ shrinks when z increases. This appears clearly in Figure 6 where we plot the optimal profits versus $c_2 = N_2/N$ for two values of z. In addition, when $X_2^* > X_1^*$, the two optimal qualities Q_1^* and Q_2^* differ significantly. Quite clearly, the lower quality targets the group of ignorant buyers while the higher quality is for informed buyers. Remarkably, when $c_2 > 0.68$, the lower optimal quality is even smaller than the optimal quality $\alpha_1/(\alpha_1 + 1) = 1/6$ corresponding to a homogeneous population of ignorant customers. This downward distortion in a situation of a monopolistic vendor is also reported in [19]; it is optimal as it reduces the substitution possibilities of higher-value (or numerous enough) customers. The benefits of low quality variants in market competition are discussed in detail in [6].

3.2 How many variants?

The phase space (c_2, z) of optimal production when at most two variants are allowed is reported in Figure 7a. At intermediate values of c_2, product differentiation exists if z is small enough (see also [14]). Can a further decrease of z differentiate further the production?

Figure 7b reveals the Russian-dolls structure of product differentiation. Let us consider the case $M = 3$: when $X_3^* > X_2^*$, in fact only two products are really different, i.e. $Q_1^* = Q_2^* < Q_3^*$. In other words, it pays to duplicate the low quality variant. This is because it decreases the likelihood that an ignorant buyer selects the high quality variant, while informed buyers, thanks to their high selection parameter, are still able to pick the premium variant. This mechanism is at work for a generic M: when z is small, for the vendor it may be optimal to display $M - 1$ low-quality variants with identical qualities and one premium variant.

Finally we consider the vendor's expected profit for various numbers of displayed variants. As can be seen in

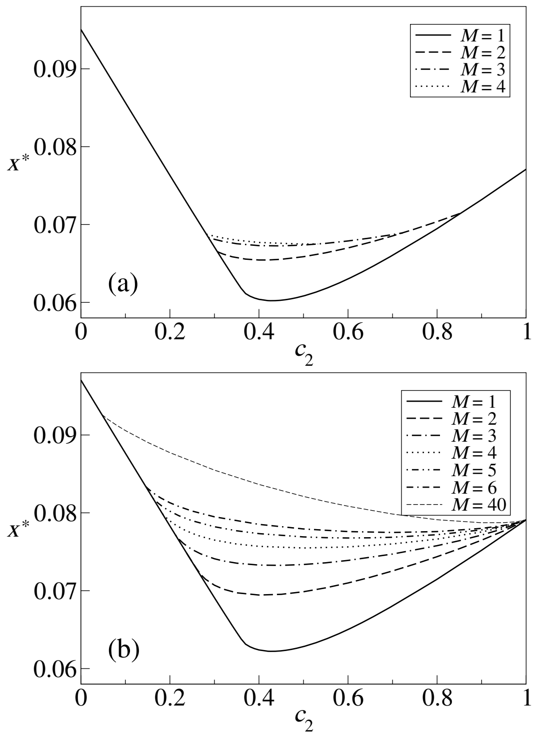

Fig. 8. Optimal profit for M displayed variants when $z = 0.002$ (a) and when $z = 0$ (b). Same parameters as in Figure 6.

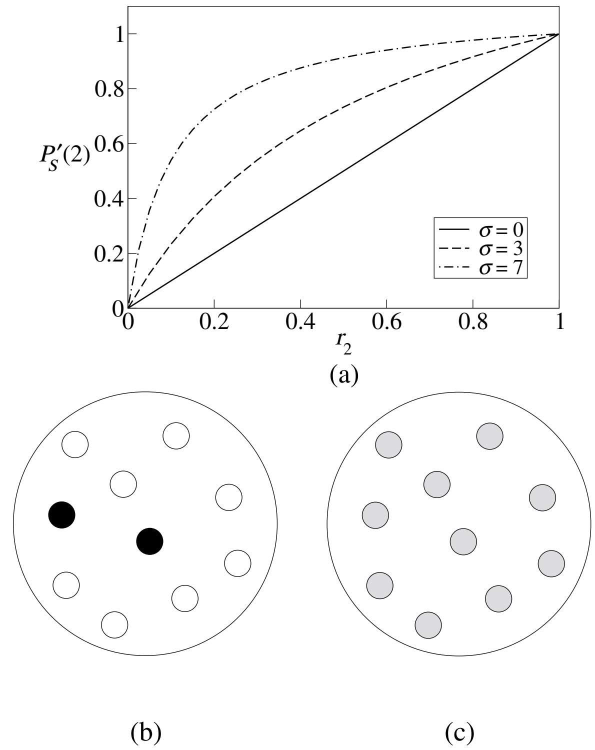

Fig. 9. (a) The probability to select variant 2 in the presence of two variants with similar qualities ($Q_1 = 0.5$, $Q_2 = 0.7$) for various σ. (b) When qualities are differentiated, informed buyers are able to select the premium variant (dark symbols) even when its proportion is small; ignorant buyers select mostly the low quality variant (white symbols). (c) When only one quality is displayed, the vendor has to compromise between the two groups and a mediocre variant is optimal (grey symbols).

Figure 8a, when $z = 0.002$, the additional gain decreases very fast when M increases and vanishes when $M > 4$. By contrast, when $z = 0$, X^* saturates at the much higher $M = 40$ and then decreases.

3.3 Biased selection

In equation (6) we implicitly assume equal standing of the available variants, which is often not the case in practice. This suggests to introduce variant weights r_m ($\sum_{m=1}^{M} r_m = 1$) in the selection probability P_S, taking into account for instance the effective visibility of each product due to advertisement or display position in shops. equation (6) generalizes to

$$P'_S(m|\boldsymbol{Q}, \boldsymbol{r}, \sigma) = \frac{r_m Q_m^\sigma}{\sum_{m'=1}^{M} r_{m'} Q_{m'}^\sigma}. \tag{8}$$

An example of the interplay between r_m and σ is shown in Figure 9: better equipped customers are able to pick the better product even when its effective proportion is small.

To study the effects of the proposed generalization we use once again two groups of customers and choose the parameters so as to set the system in the quality differentiation region. Results of numerical optimization of the optimal profit are reported in Figure 10, r_2 denotes the proportion of the premium variant. Differentiation occurs in a limited range of r_2: when r_2 is either too small or too large, buyers effectively notice only one variant and it is preferable for the vendor to produce only that one. In addition, x^* has a maximum at $r_2 \approx 0.08$, which comes from hiding the high quality variant to ignorant buyers while keeping it accessible to informed buyers.

4 Price

Let us now consider the price as a free parameter and investigate how the vendor should fix it optimally. Denoting the price by p, the profit per item is $p - Q$ which means that the maximum quality is $Q_{\max} = p$. In particular, if the vendor wishes to produce a better product that $Q_{\max}$, the price needs to be increased. The acceptance probability generalizes to ($\alpha \geq 0,\ p \in [Q, \alpha + 1]$)

$$P_A(Q, p, \alpha) = \left(1 - \frac{p}{\alpha + 1}\right)\left(\frac{Q}{p}\right)^\alpha. \tag{9}$$

It satisfies two constraints: first, the higher the price, the smaller the acceptance probability. Second, because of the $p/(\alpha+1)$ term, the sensitivity towards prices decreases as sensitivity to quality increases; similarly, quality must be judged with respect to price, hence the $(Q/p)^\alpha$ term. The discussion of the previous sections corresponds to $p = 1$.

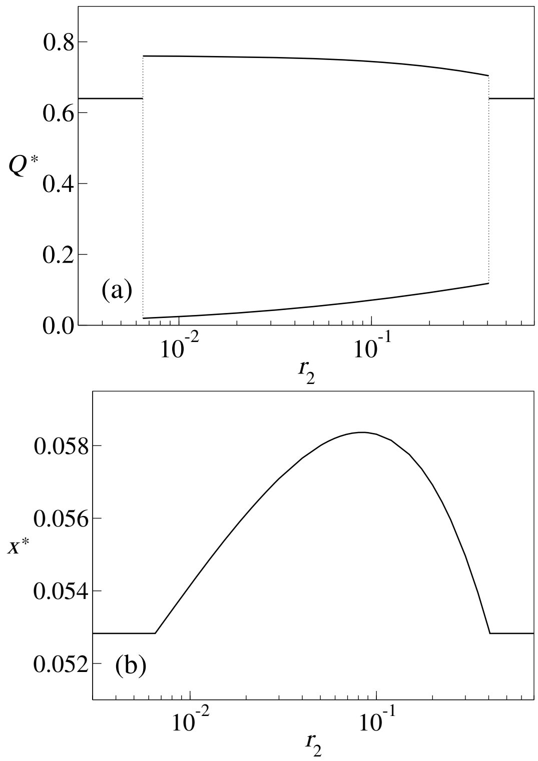

Fig. 10. Optimal qualities (a) and the optimal profit (b) versus proportion of the premium variant r_2 ($\alpha_1 = 0.2$, $\alpha_2 = 3$, $\sigma_1 = 0.2$, $\sigma_2 = 2$, $z = 0.01$, $c_2 = 0.5$, $M = 2$).

We restrict our analysis to the simplest case of N identical buyers and one product. The expected profit reads

$$X_1(Q,p) = N(p-Q)\left(1 - \frac{p}{\alpha+1}\right)\left(\frac{Q}{p}\right)^{\alpha}$$

with $p \leq \alpha + 1$ and $Q \in [0,p]$, it is maximized by

$$Q^* = \frac{\alpha}{2}, \quad p^* = \frac{\alpha+1}{2}. \tag{10}$$

Expectedly, the more informed the buyers, the better the products should be, but the vendor can charge a higher price. Because $p^* - Q^* = 1/2$ is a constant, there is no incentive in this model for exceptionally high prices for high quality variants. In Figure 11, the resulting optimal profit per buyer x^* is shown together with the optimal profit when the vendor has fixed the price at 1. The liberty to set the price can increase the profit of the vendor quite considerably. The difference of profit for $\alpha > 1$ (informed buyers) is due to the fact that the vendor is allowed to charge a higher price for the high quality demanded by the buyers. By contrast, for $\alpha < 1$ the main improvement comes from the fact that $P_A(Q,p,\alpha)$ does not vanish when $Q \to 0$ and $p < 1$.

5 Conclusion

Due to the complexity of markets and human behaviour, attempts to propose a theory of the whole are illusory.

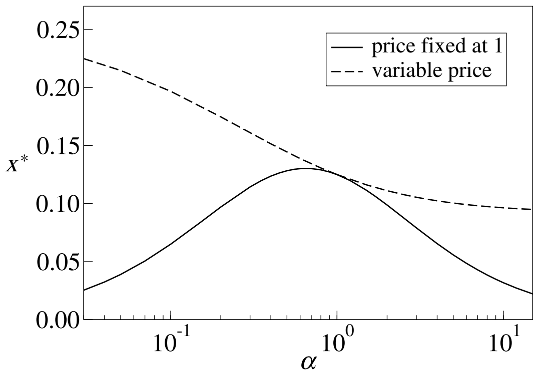

Fig. 11. Optimal profit x^* vs acceptance parameter α: fixed price (solid line, the same curve as in Fig. 2) and variable price (dashed line), $z = 0$.

However, simple models can bring insight to elementary mechanisms at work in the real economy. Assuming probabilistic buyer behaviour, we formalized buyers' abilities, spanning from the zero information to the perfect information limits. Adopting the vendor's point of view, we examined the compromise between low quality which minimizes production costs and high quality which maximizes sales. In particular, the fact that customers are heterogeneous forces vendors to diversify their production. In other words, the large variety of products in free-market economies reflects in part the information gathering and processing abilities of customers.

In this work we focused on the basic market phenomena but the proposed model is versatile enough to represent more complicated cases. Three important extensions seem particularly worth further investigation: including explicitly the price in heterogeneous populations, generalizing the present results to an arbitrary number of consumer groups, and adding more vendors and letting them to compete for customers (then it can be Nash stable that variants with different qualities are provided by different vendors).

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Appendix A: Discussion of the model plausibility

In order to better understand the need for both selection and acceptance procedures, it is worthwhile to consider some alternatives. One possibility to simplify our assumptions is to keep only the acceptance process with each displayed variant accepted or not according to the acceptance probability P_A. When M variants with the qualities

$Q_1, \ldots, Q_M$ are displayed, the probability P' that a given customer accepts at least one of them is

$$P'(Q_1, \ldots, Q_M) = 1 - \prod_{a=1}^{M} \big[1 - P_A(Q_a)\big]. \tag{11}$$

As M increases, P' converges to one. This means that the vendor can attract the buyers by displaying a large number of very bad products which is generally not the case. However, flooding of customers by low quality occurs under some special circumstances. This *economics of spamming* is briefly discussed in the next Appendix.

Another approach is to reduce the model to the best product selection governed by the selection probability P_S. Since this probability is normalized to one, when it is applied alone, each buyer surely buys one of the displayed variants and consequently the vendor's profit maximization yields zero quality. Obviously, such an optimal solution is pathological. One could eventually consider replacing the unity in the equation $\sum_{m=1}^{M} P_S(m|\boldsymbol{Q}, \sigma) = 1$ by an increasing function of the displayed qualities but this is effectively equivalent to our two step decision process. Another possibility is to introduce an artificial non-purchase alternative to equation (6) with an imposed utility as in [13] which focuses on price differentiation. We see that nor the selection from the available variants is sufficient to model the purchase process.

Finally, the generalization to diverse proportions of displayed variants, introduced in Section 3.3, gives an additional argument. We see that while in the selection step both quality and proportion play their roles, in the final acceptance step it is only quality of the selected variant what matters. Thus these two steps are intrinsically different and attempts to merge them are artificial.

Appendix B: Economics of spamming

By the economics of spamming we understand the situation when a low initial cost Z allows the vendor to produce an abundance of low quality variants. We simplify our considerations to M variants with identical qualities Q and identical buyers with acceptance parameter α. Assuming that the variants are displayed consecutively, the selection probability plays no role. This situation resembles spam messages arriving into our mailboxes which we reject one by one. According to equation (11), on average $N(1-[1-P_A(Q,\alpha)]^M)$ buyers accept one of the displayed variants and the expected vendor profit per buyer is therefore

$$x(Q, M) = (1-Q)\Big(1 - \big[1 - P_A(Q,\alpha)\big]^M\Big) - Mz. \tag{12}$$

Since we focus on low quality variants, $P_A(Q,\alpha)$ is small and the approximation $1 - x \approx \exp[-x]$ can be used. It follows that for a given quality Q, the optimal number of displayed variants is

$$M^*(Q) = \frac{\alpha+1}{\alpha Q^\alpha} \ln \frac{\alpha(1-Q)Q^\alpha}{z(\alpha+1)}\cdot \tag{13}$$

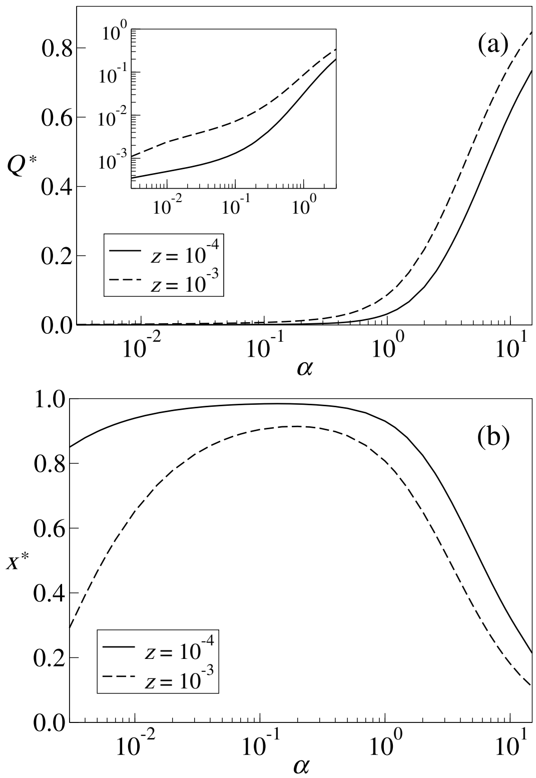

Fig. 12. Optimal quality (a) and optimal vendor profit per buyer (b) versus acceptance parameter α.

However, when buyers' perception is limited to a certain number of variants M_m, this value applies instead of $M^*(Q)$. Assuming small initial cost z, the leading contribution to the optimal quality can be shown to verify $b\ln[\alpha Q^\alpha/b] = Q^{\alpha+1}$, where $b := z(1+1/\alpha)$). When the resulting $Q^* \ll 1$, it follows that equation (13) take the simple form $M^*(Q^*) = Q^*/(\alpha z)$; the optimal quality Q^* has to be found numerically. The results are shown in Figure 12: as z gets lower, the optimal quality decreases and the optimal profit increases; in addition, when acceptance parameters are small ($10^{-2} \lesssim \alpha \lesssim 1$) the optimal profit per buyer is almost one which means that nearly all buyers react to the spamming.

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